

Quantum correlations among optical and vibrational quanta

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We investigate the feasibility of correlating an optical cavity field and a vibrational phonon mode. A laser pumped quantum dot fixed on a nano-mechanical resonator beam interact as a whole with the optical resonator mode. When the quantum dot variables are faster than the optical and phonon ones, we obtain a final master equation describing the involved modes only. Increasing the temperature, that directly affects the vibrational degrees of freedom, one can as well influence the cavity photon intensity, i.e., the optical and phonon modes are correlated. Furthermore, the corresponding Cauchy-Schwarz inequality is violated demonstrating the quantum nature of those correlations.

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I. INTRODUCTION

Nano-mechanical resonators are actively and widely investigated [1, 2]. Their importance for ultra-light mass detection, ultra-small displacements or very small force measurements is now well recognized [2] (and references therein). Significant contributions were achieved towards macroscopic quantum effects in these systems combined with existing ones. For instance, superconducting qubit storage and entanglement with nano-mechanical resonators was investigated in [3] while feedback-enhanced parametric squeezing of mechanical motion was investigated in Ref. [4], respectively. Observation of quantum motion of a nano-mechanical resonator was reported in [5]. Coherent phonon manipulation in coupled mechanical resonators was experimentally demonstrated in [6] and this may help in entangling two distinct macroscopic mechanical objects [7]. A nano-mechanical interface between optical photons and microwave electrical signals was recently demonstrated as well [8]. Furthermore, the dipole-dipole coupling among a small ensemble of Rydberg atoms allows for quantum control of macroscopic mechanical oscillators via their mutual interactions [9] whereas stationary continuous-variable entanglement between an optical cavity and a nano-mechanical resonator beam was demonstrated too, in Ref. [10]. Finally, phonon lasing was experimentally achieved in an electromechanical resonator [11].

Here, we investigate a system composed of a two-level quantum dot fixed on a suspended nano-mechanical resonator inside an optical cavity. The quantum dot is externally laser pumped. Spontaneous decay as well as optical or vibrational modes damping rates are correspondingly taking into account. We are interested in a regime where the quantum dot dynamics is faster than the dynamics of other subsystems involved, i.e. in the good cavity limit. In this way, one can reduce the whole system to the quantum dynamics of the two modes, vibrational

and optical. We found quantum correlations among the optical and vibrational quanta under particular circumstances of stronger qubit-resonator coupling strengths. Actually, we have demonstrated the violation of the Cauchy-Schwarz inequality defined as a product of the photon-photon and phonon-phonon second-order correlation functions divided on the photon-phonon or phonon-photon cross-correlation function squared. Moreover, the environmental temperature that explicitly affects the mechanical subsystem only modifies as well the mean-photon number in the optical resonator mode demonstrating the existence of correlations among the photon and phonon modes.

The article is organized as follows. In Sec. II we describe the model as well as the analytical approach used. The master equation characterizing the induced correlations among optical and phonon modes is derived there. Sec. III deals with the corresponding equations of motion and discussion of the obtained results. The Summary is given in the last section, i.e. Sec. IV.

II. ANALYTICAL APPROACH

The investigated model is described as follows: A two-level quantum dot of frequency ω_0 is fixed on a semiconductor beam structure inside an optical cavity. A coherent laser field with the wave-vector \vec{k}_L is resonantly interacting with the two-level quantum dot leading to correlations between mechanical vibrations and photon scattering. If the thickness of the beam is smaller than its width, the lowest-energy resonance corresponds to the fundamental flexural mode with the frequency being of the order of GHz. Flexions induce extensions and compressions in the structure modifying the deformation potential coupling of the embedded quantum dot and, hence, its energy levels [12]. Concomitantly, the artificial two-level emitter interacts with the optical cavity mode. The vibrational and optical mode frequencies are denoted by ω and ω_c , respectively. The model Hamiltonian de-

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scribing the whole system is:

$$H = \hbar\omega_c a^\dagger a + \hbar\omega_b b^\dagger b + \hbar\omega_0 S_z + \hbar g(a^\dagger S^- + a S^+) + \hbar\Omega(S^+ e^{-i\omega_L t} + S^- e^{i\omega_L t}) + \hbar\lambda S_z(b^\dagger + b). \quad (1)$$

Here, the first three terms describe the free energies of optical and mechanical modes as well as of the artificial two-level system. The fourth and the fifth terms characterize the interaction of the quantum dot with the optical resonator mode and laser field, respectively. The last term takes into account the interaction of the vibrational degrees of freedom with the radiator [12]. g and λ denote the interaction strengths among the two-level emitter and the involved optical and mechanical modes, while Ω is the corresponding Rabi frequency due to external laser pumping. The qubit operators S_z and S^\pm have the usual meaning and satisfy the standard commutation relations. $\{a^\dagger, b^\dagger\}$ and $\{a, b\}$ are the generation and annihilation operators for photon and phonon subsystems, respectively, and obey the boson commutation relations [13, 14].

The entire system will be described in the dressed-state representation [14, 15]: $|g\rangle = \sin\theta|+\rangle + \cos\theta|-\rangle$ and $|e\rangle = \cos\theta|+\rangle - \sin\theta|-\rangle$, where $\cot 2\theta = \Delta/2\Omega$ with $\Delta = \omega_0 - \omega_L$ being the detuning of the laser frequency ω_L from the two-level transition frequency. $|e\rangle$ and $|g\rangle$ are the excited and the ground bare states of the quantum dot, while $|+\rangle$ and $|-\rangle$ the corresponding states in the dressed-state picture. In the interaction picture, the master equation describing our model in the rotating-wave and dipole approximations as well as in Born-Markov approximations is:

$$\begin{aligned} \dot{\rho} + \frac{i}{\hbar}[H_d, \rho] = & -\gamma_0[R_z, R_z\rho] - \gamma_+[R_\pm, R_\mp\rho] \\ & - \gamma_-[R_\mp, R_\pm\rho] - \kappa_a[a^\dagger, a\rho] - \kappa_b(1 + \bar{n})[b^\dagger, b\rho] \\ & - \kappa_b\bar{n}[b, b^\dagger\rho] + H.c., \end{aligned} \quad (2)$$

where an overdot denotes differentiation with respect to time. Here, the dressed-state Hamiltonian is $H_d = H_0 + H_i$ with:

$$\begin{aligned} H_0 &= \hbar\Omega_R R_z - \hbar\Delta_1 a^\dagger a + \hbar\omega_b b^\dagger b, \\ H_i &= AR_z + B^\dagger R_\mp e^{-2i\Omega_R t} + BR_\pm e^{2i\Omega_R t}, \end{aligned} \quad (3)$$

where $\Delta_1 = \omega_L - \omega_c$ and $\Omega_R = \sqrt{(\Delta/2)^2 + \Omega^2}$, whereas

$$\begin{aligned} A &= \frac{\hbar}{2}(g(ae^{i\Delta_1 t} + a^\dagger e^{-i\Delta_1 t})\sin 2\theta \\ &\quad + \lambda(be^{-i\omega t} + b^\dagger e^{i\omega t})\cos 2\theta), \\ B &= \hbar g(ae^{i\Delta_1 t}\cos^2\theta - a^\dagger e^{-i\Delta_1 t}\sin^2\theta) \\ &\quad - \frac{\hbar}{2}\lambda\sin 2\theta(be^{-i\omega t} + b^\dagger e^{i\omega t}). \end{aligned} \quad (4)$$

The dressed-state quantum dot operators are defined as follows: $R_z = |+\rangle\langle+| - |-\rangle\langle-|$, $R_\pm = |+\rangle\langle-|$ and $R_\mp = |-\rangle\langle+|$ satisfying the standard commutation relations for su(2) algebra. Further, $\gamma_0 = \frac{1}{4}(\gamma\sin^2 2\theta + \gamma_c\cos^2 2\theta)$,

$\gamma_+ = \gamma\cos^4\theta + \frac{\gamma_c}{4}\sin^2 2\theta$ and $\gamma_- = \gamma\sin^4\theta + \frac{\gamma_c}{4}\sin^2 2\theta$ describe the spontaneous decay processes among the involved dressed-states, while γ and γ_c are the single-qubit spontaneous decay and dephasing rates, respectively. $\kappa_a(\kappa_b)$ is the photon (phonon) resonator damping rate whereas \bar{n} is the mean-phonon number corresponding to vibrational frequency ω and environmental temperature T .

The master equation (2) is quite complex. However, for our purposes one can significantly simplify it. Particularly, we are interested in a regime where the pumped quantum-dot system is faster than the cavity photon and vibrational phonon subsystems, respectively. Consequently, the quantum dot variables can be eliminated from the whole quantum dynamics, an approximation valid for $\Omega \gg \gamma \gg \kappa_{a,b}$ as well as $\Omega \gg \{g, \lambda\}$. Notice that this approach is widely used in other somehow related systems [16–19]. In the following, we write down the master equation (2) after tracing over quantum-dot degrees of freedom:

$$\begin{aligned} \dot{\rho}_f = & -i[A, \rho_{++} - \rho_{--}] - i[B^\dagger, \rho_{+-}]e^{-2i\Omega_R t} \\ & - i[B, \rho_{-+}]e^{2i\Omega_R t} - \kappa_a[a^\dagger, a\rho_f] \\ & - \kappa_b(1 + \bar{n})[b^\dagger, b\rho_f] - \kappa_b\bar{n}[b, b^\dagger\rho_f] + H.c., \end{aligned} \quad (5)$$

where $\rho_{\alpha\beta} = \langle\alpha|\rho_q|\beta\rangle\rho_f$, with $\{\alpha, \beta \in \pm\}$. The quantum dot variables can be found from Eq. (2), namely:

$$\begin{aligned} \dot{\rho}_{+-} = & -\Gamma_\perp\rho_{+-} - i(A\rho_{+-} + \rho_{+-}A) \\ & - i(B\rho_{--} - \rho_{++}B)e^{2i\Omega_R t}, \\ \dot{\rho}_{++} = & -2\gamma_+\rho_{++} + 2\gamma_-\rho_{--} - i(A\rho_{++} - \rho_{++}A) \\ & - i(B\rho_{-+}e^{2i\Omega_R t} - \rho_{+-}B^\dagger e^{-2i\Omega_R t}), \\ \dot{\rho}_{--} = & 2\gamma_+\rho_{++} - 2\gamma_-\rho_{--} + i(A\rho_{--} - \rho_{--}A) \\ & + i(\rho_{-+}Be^{2i\Omega_R t} - B^\dagger\rho_{+-}e^{-2i\Omega_R t}), \end{aligned} \quad (6)$$

with $\Gamma_\perp = 4\gamma_0 + \gamma_+ + \gamma_-$. In the secular approximation and to first order in the interaction parameters $\{g, \lambda\}$ the solutions of (6) are:

$$\begin{aligned} \rho_{+-} &= -ie^{2i\Omega_R t}(\bar{B}\rho_{--} - \rho_{++}\bar{B}), \\ \rho_{++} - \rho_{--} &= -i(\bar{A}\rho_f - \rho_f\bar{A}), \end{aligned} \quad (7)$$

where

$$\begin{aligned} \bar{A} &= \frac{g}{2}\left(\frac{a^\dagger e^{-i\Delta_1 t}}{\Gamma_\parallel - i\Delta_1} + \frac{ae^{i\Delta_1 t}}{\Gamma_\parallel + i\Delta_1}\right)\sin 2\theta \\ &\quad + \frac{\lambda}{2}\left(\frac{b^\dagger e^{i\omega t}}{\Gamma_\parallel + i\omega} + \frac{be^{-i\omega t}}{\Gamma_\parallel - i\omega}\right)\cos 2\theta, \\ \bar{B} &= -\frac{\lambda}{2}\left(\frac{\sin 2\theta b^\dagger e^{i\omega t}}{\Gamma_\perp + i(2\Omega_R + \omega)} + \frac{\sin 2\theta be^{-i\omega t}}{\Gamma_\perp + i(2\Omega_R - \omega)}\right) \\ &\quad + g\left(\frac{a\cos^2\theta e^{i\Delta_1 t}}{\Gamma_\perp + i(2\Omega_R + \Delta_1)} - \frac{a^\dagger\sin^2\theta e^{-i\Delta_1 t}}{\Gamma_\perp + i(2\Omega_R - \Delta_1)}\right). \end{aligned}$$

Here $\Gamma_\parallel = \gamma(1 + \cos^2 2\theta) + \gamma_c\sin^2 2\theta$. Inserting the solutions given by (7) in Eq. (5) and using the identity $\text{Tr}\{\dot{\rho}(t)Q\} = \text{Tr}\{\dot{Q}(t)\rho\}$, one can obtain the final master

equation describing the quantum dynamics of the photon and phonon subsystems in the good cavity limit:

$$\begin{aligned} \langle \dot{Q} \rangle + \frac{i}{2}(\Delta_1 - \omega)\langle [a^\dagger a + b^\dagger b, Q] \rangle = \\ = \langle [Q, a](-A_1^* a^\dagger + D_2^* b) + (B_1^* a^\dagger - C_2^* b)[Q, a] \rangle \\ + \langle [Q, a^\dagger](-B_1 a + C_2 b^\dagger) + (A_1 a - D_2 b^\dagger)[Q, a^\dagger] \rangle \\ + \langle [Q, b](D_1^* a - A_2^* b^\dagger) + (B_2^* b^\dagger - C_1^* a)[Q, b] \rangle \\ + \langle [Q, b^\dagger](C_1 a^\dagger - B_2 b) + (A_2 b - D_1 a^\dagger)[Q, b^\dagger] \rangle. \end{aligned} \quad (8)$$

Here Q denotes, respectively, any operator belonging to the photon and phonon subsystems, while

$$\begin{aligned} A_1 &= \frac{1}{4} \frac{g^2 \sin^2 2\theta}{\Gamma_{\parallel} + i\Delta_1} + \frac{g^2 P_- \sin^4 \theta}{\Gamma_{\perp} - i(2\Omega_R - \Delta_1)} \\ &+ \frac{g^2 P_+ \cos^4 \theta}{\Gamma_{\perp} + i(2\Omega_R + \Delta_1)}, \\ A_2 &= \frac{1}{4} \left(\frac{\lambda^2 \cos^2 2\theta}{\Gamma_{\parallel} - i\omega} + \frac{\lambda^2 P_- \sin^2 2\theta}{\Gamma_{\perp} - i(2\Omega_R + \omega)} \right. \\ &\left. + \frac{\lambda^2 P_+ \sin^2 2\theta}{\Gamma_{\perp} + i(2\Omega_R - \omega)} \right) + \kappa_b \bar{n}, \\ C_1 &= \frac{P_+}{2} \frac{g\lambda \sin 2\theta \cos^2 \theta}{\Gamma_{\perp} - i(2\Omega_R + \Delta_1)} - \frac{P_-}{2} \frac{g\lambda \sin 2\theta \sin^2 \theta}{\Gamma_{\perp} + i(2\Omega_R - \Delta_1)} \\ &- \frac{1}{4} \frac{g\lambda \sin 2\theta \cos 2\theta}{\Gamma_{\parallel} - i\Delta_1}, \\ C_2 &= \frac{P_-}{2} \frac{g\lambda \sin 2\theta \cos^2 \theta}{\Gamma_{\perp} + i(2\Omega_R + \omega)} - \frac{P_+}{2} \frac{g\lambda \sin 2\theta \sin^2 \theta}{\Gamma_{\perp} - i(2\Omega_R - \omega)} \\ &- \frac{1}{4} \frac{g\lambda \sin 2\theta \cos 2\theta}{\Gamma_{\parallel} + i\omega}, \end{aligned}$$

with

$$P_+ = \frac{\gamma_-}{\gamma_+ + \gamma_-}, \quad \text{and} \quad P_- = \frac{\gamma_+}{\gamma_+ + \gamma_-}.$$

B_i can be obtained from A_i via $P_{\mp} \leftrightarrow P_{\pm}$ as well as by adding κ_a to B_1 and κ_b to B_2 , correspondingly. Respectively, D_i can be obtained from C_i through $P_{\mp} \leftrightarrow P_{\pm}$, and $\{i \in 1, 2\}$. Notice that in obtaining Eq. (8) we have ignored rapidly oscillating terms at frequencies: $\pm 2\Delta_1$, $\pm(\Delta_1 + \omega)$ and $\pm 2\omega$. Actually, we are interesting in a regime where a laser photon absorption is accompanied by the generation of a phonon and an optical cavity photon, respectively, that is when $\Delta_1 \approx \omega$.

In the following section, we shall describe the induced quantum correlations among the optical and mechanical degrees of freedom.

III. QUANTUM CORRELATIONS

Eq. (8) allows us to obtain the equations of motion for the variables of interest. We can define $Q = a^{\dagger j} a^k b^{\dagger l} b^m$ (where $\{j, k, l, m\}$ are any integer numbers) as a general operator belonging to the both, photon and phonon subsystems. The equation of motions for the mean-values of

photon and phonon numbers etc. can be then obtained from the following main equation:

$$\begin{aligned} \frac{d}{dt} \langle a^{\dagger j} a^k b^{\dagger l} b^m \rangle = \langle a^{\dagger j} a^k b^{\dagger l} b^m \rangle \\ \times ((A_1^* - B_1^*)j + (A_1 - B_1)k + (A_2^* - B_2^*)l \\ + (A_2 - B_2)m - \frac{i}{2}(\Delta_1 - \omega)(j - k + l - m)) \\ + \langle a^{\dagger j+1} a^k b^{\dagger l} b^{m-1} \rangle (C_1 - D_1)m \\ + \langle a^{\dagger j-1} a^k b^{\dagger l} b^{m+1} \rangle (C_2^* - D_2^*)j \\ + \langle a^{\dagger j} a^{k+1} b^{\dagger l-1} b^m \rangle (C_1^* - D_1^*)l \\ + \langle a^{\dagger j} a^{k-1} b^{\dagger l+1} b^m \rangle (C_2 - D_2)k \\ + \langle a^{\dagger j-1} a^k b^{\dagger l-1} b^m \rangle (C_1^* + C_2^*)jl \\ + \langle a^{\dagger j-1} a^{k-1} b^{\dagger l} b^m \rangle (A_1 + A_1^*)jk \\ + \langle a^{\dagger j} a^{k-1} b^{\dagger l} b^{m-1} \rangle (C_1 + C_2)km \\ + \langle a^{\dagger j} a^k b^{\dagger l-1} b^{m-1} \rangle (A_2 + A_2^*)lm. \end{aligned} \quad (9)$$

For instance, by selecting instead of $\{j, k, l, m\}$ the following sets: $\{1, 1, 0, 0\}$, $\{0, 0, 1, 1\}$, $\{0, 1, 0, 1\}$, $\{1, 0, 1, 0\}$ one arrives at the equations of motion for the mean-values of the photon and phonon numbers and their first-order correlations, namely:

$$\begin{aligned} \frac{d}{dt} \langle a^\dagger a \rangle &= \langle a^\dagger a \rangle (A_1 - B_1 + A_1^* - B_1^*) + \langle ab \rangle (C_2^* - D_2^*) \\ &+ \langle a^\dagger b^\dagger \rangle (C_2 - D_2) + A_1 + A_1^*, \\ \frac{d}{dt} \langle b^\dagger b \rangle &= \langle b^\dagger b \rangle (A_2 - B_2 + A_2^* - B_2^*) + \langle ab \rangle (C_1^* - D_1^*) \\ &+ \langle a^\dagger b^\dagger \rangle (C_1 - D_1) + A_2 + A_2^*, \\ \frac{d}{dt} \langle ab \rangle &= \langle ab \rangle (A_1 - B_1 + A_2 - B_2 + i(\Delta_1 - \omega)) \\ &+ \langle a^\dagger a \rangle (C_1 - D_1) + \langle b^\dagger b \rangle (C_2 - D_2) + C_1 + C_2, \\ \frac{d}{dt} \langle a^\dagger b^\dagger \rangle &= \langle a^\dagger b^\dagger \rangle (A_1^* - B_1^* + A_2^* - B_2^* - i(\Delta_1 - \omega)) \\ &+ \langle a^\dagger a \rangle (C_1^* - D_1^*) + \langle b^\dagger b \rangle (C_2^* - D_2^*) + C_1^* + C_2^*. \end{aligned} \quad (10)$$

Additionally, the equations of motion for the second-order correlation functions and their cross-correlations, that is:

$$\begin{aligned} g_1^{(2)}(0) &= \frac{\langle a^\dagger a^\dagger aa \rangle}{\langle a^\dagger a \rangle^2}, \quad g_2^{(2)}(0) = \frac{\langle b^\dagger b^\dagger bb \rangle}{\langle b^\dagger b \rangle^2}, \\ g_3^{(2)}(0) &= \frac{\langle a^\dagger ab^\dagger b \rangle}{\langle a^\dagger a \rangle \langle b^\dagger b \rangle}, \end{aligned} \quad (11)$$

can be obtained again from Eq. (9) by considering, respectively, the next sets of numbers: $\{2, 2, 0, 0\}$, $\{0, 0, 2, 2\}$, $\{1, 1, 1, 1\}$, $\{1, 0, 2, 1\}$, $\{0, 1, 1, 2\}$, $\{2, 1, 1, 0\}$, $\{1, 2, 0, 1\}$, $\{2, 0, 2, 0\}$, $\{0, 2, 0, 2\}$ instead of $\{j, k, l, m\}$. An efficient tool to investigate the quantum features of the correlations between photons and phonons is the Cauchy-Schwarz inequality, CSI, which is defined as follows [20]:

$$\text{CSI} = g_1^{(2)}(0)g_2^{(2)}(0)/[g_3^{(2)}(0)g_3^{(2)}(0)]. \quad (12)$$

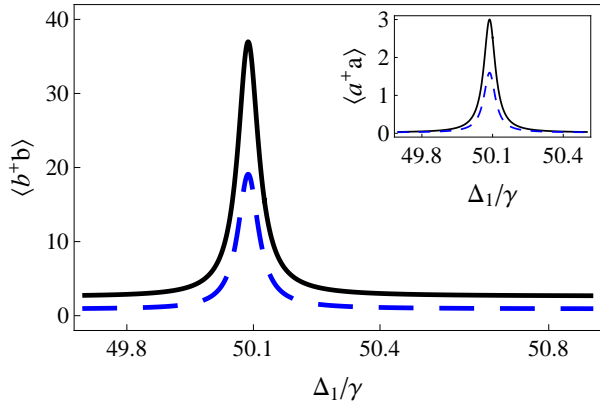


FIG. 1: (color online) The mean-value of the phonon number $\langle b^\dagger b \rangle$ as a function of Δ_1/γ . Here, $\gamma_c/\gamma = 0.3$, $g/\gamma = 3$, $\lambda/\gamma = 5$, $\Omega/\gamma = 50$, $\omega/\gamma = 50$, $\Delta/(2\Omega) = -0.263$, $\kappa_a/\gamma = 0.09$ and $\kappa_b/\gamma = 0.009$. The solid line corresponds to $\bar{n} = 2$, whereas the dashed one to $\bar{n} = 0.5$. The inset figure shows the same but for the photon mean-number $\langle a^\dagger a \rangle$, respectively.

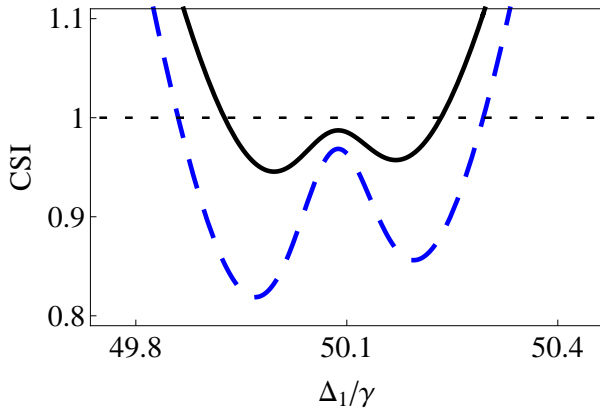


FIG. 2: (color online) The steady-state value of the CSI versus Δ_1/γ . The solid line corresponds to $\bar{n} = 2$, whereas the long-dashed one to $\bar{n} = 0.5$. Violation of the CSI occurs below the horizontal dashed line. Other parameters are the same as in Fig. (1).

Particularly, the induced correlations are of quantum nature if $\text{CSI} < 1$.

Figure (1) shows the steady-state value of the mean phonon and photon numbers for particular parameters close to those already considered in other setups [1, 2, 10, 12, 15, 21]. For stronger qubit-cavity couplings one can observe a peak of these quantities around

$\Delta_1 \approx \omega$, i.e. where a laser photon absorption is accompanied by the generation of a phonon and an optical cavity photon, respectively, while $\Delta_1 \neq \Delta$. A frequency shift occurring due to nonlinear effects existing in the dispersive limit applied here (see, also, [16–19]) is responsible for the resonance around ω and not exactly at this value. Furthermore, the environment temperature that explicitly influences the vibrational degrees of freedom, implicitly affects also the mean photon number in the optical resonator mode (compare solid and dashed curves) demonstrating, thus, the existence of correlations among them. The quantum nature of these correlations can be demonstrated via the violation of the CSI given by Eq. (12). Therefore, the steady-state behaviors of the CSI versus Δ_1/γ is shown in Fig. (2). The CSI is violated, i.e. $\text{CSI} < 1$, near $\Delta_1 \approx \omega$ where the mean values of the photon and phonon numbers are maximal. Notice that quantum features of these correlations will disappear for $|\Delta_1 - \omega| \gg \gamma$. This can be seen also from the master equation (8) where the terms responsible for the cross-correlations among the two different modes will simply vanish in this regime, i.e. when $|\Delta_1 - \omega|/\gamma \gg 1$.

IV. SUMMARY

In summary, we have studied the quantum nature of induced correlations among mechanical and optical degrees of freedom. A two-level quantum dot leads to correlated vibrations when fixed on a nano-mechanical resonator beam while interacting with an external coherent laser field as well as with an optical mode of a leaking resonator. When the variables describing the pumped quantum dot are faster than those of other involved subsystems and the detuning of the laser frequency from the cavity one is located around the vibrating mode frequency, we have found quantum correlations between the photon and phonon subsystems. Actually, stronger qubit-cavity coupling strengths are required that can be achieved also via involving more two-level artificial qubits.

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- [1] X. Huang, C. Zorman, M. Mehregany, M. Roukes, *Nature* **421**, 496 (2003).
 - [2] Y. Greenberg, Y. Pashkin, E. Il'ichev, *Physics-Uspekhi* **55**, 382 (2012).
 - [3] A. Cleland, M. Geller, *Phys. Rev. Lett.* **93**, 070501 (2004).

- [4] A. Vinante, P. Falferi, *Phys. Rev. Lett.* **111**, 207203 (2013).
- [5] A. Safavi-Naeini, J. Chan, J. Hill, T. Mayer Alegre, A. Krause, O. Painter, *Phys. Rev. Lett.* **108**, 033602 (2012).
- [6] H. Okamoto, A. Gourgout, C. Chang, K. Onomitsu, I. Mahboob, E. Chang, H. Yamaguchi, *Nature Physics* **9**,

- 480 (2013).
- [7] K. Brown, C. Ospelkaus, Y. Colombe, A. C. Wilson, D. Leibfried, D. Wineland, *Nature* **471**, 196 (2011).
 - [8] J. Bochmann, A. Vainsencher, D. Awschalom, A. Cleland, *Nature Physics* **9**, 712 (2013).
 - [9] F. Bariani, J. Otterbach, H. Tan, P. Meystre, *Phys. Rev. A* **89**, 011801(R) (2014).
 - [10] X.-z. Yuan, *Phys. Rev. A* **88**, 052317 (2013).
 - [11] I. Mahboob, K. Nishiguchi, A. Fujiwara, H. Yamaguchi, *Phys. Rev. Lett.* **110**, 127202 (2013).
 - [12] I. W. Rae, P. Zoller, A. Imamoglu, *Phys. Rev. Lett.* **92**, 075507 (2004).
 - [13] M. Kiffner, M. Macovei, J. Evers, C. H. Keitel, *Prog. Opt.* **55**, 85 (2010).
 - [14] J. Peng, G.-x. Li, *Introduction to Modern Quantum Optics* (World Scientific, 1998).
 - [15] S. Das, M. A. Macovei, *Phys. Rev. B* **88**, 125306 (2013).
 - [16] W. Ge, M. Al-Amri, H. Nha, M. S. Zubairy, *Phys. Rev. A* **88**, 022338 (2013); *Phys. Rev. A* **88**, 052301 (2013).
 - [17] H. Xiong, M. O. Scully, M. S. Zubairy, *Phys. Rev. Lett.* **94**, 023601 (2005); H. T. Tan, S. Y. Zhu, M. S. Zubairy, *Phys. Rev. A* **72**, 022305 (2005).
 - [18] M. Kiffner, M. S. Zubairy, J. Evers, C. H. Keitel, *Phys. Rev. A* **75**, 033816 (2007).
 - [19] Z.-h. Tang, G.-x. Li, *Phys. Rev. A* **84**, 063801 (2011); Z.-h. Tang, G.-x. Li, Z. Ficek, *Phys. Rev. A* **82**, 063837 (2010); G.-x. Li, H.-t. Tan, M. Macovei, *Phys. Rev. A* **76**, 053827 (2007).
 - [20] J. F. Clauser, *Phys. Rev. D* **9**, 853 (1974); R. Loudon, *Rep. Prog. Phys.* **43**, 58 (1980).
 - [21] J.-J. Li, K.-D. Zhu, *Phys. Rev. B* **83**, 245421 (2011).